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Instabilities in a Magnetized Beam-Plasma System

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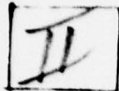
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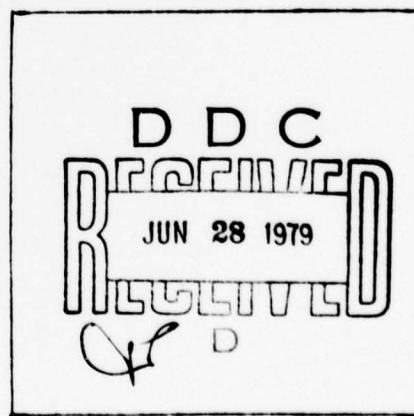
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ABSTRACT

A review of the existing theory on instabilities in a magnetized beam-plasma system is presented with emphasis on the unstable electromagnetic waves in the frequency range between ion cyclotron frequency and the electron cyclotron frequency. The strengths and weaknesses of the various theoretical models are discussed and it is pointed out in which direction future work should proceed. Both the unbounded and radially bounded problems are considered, using both a quasistatic treatment and a fully electromagnetic treatment. Plots of the dispersion curves and a table of the quasistatic growth rates for four of the instabilities are given for reference.

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Introduction

The use of relativistic electron beams for heating a plasma to thermonuclear temperatures is presently under extensive investigation. It has been shown that binary collisions can have little effect on plasma heating so that collective effects are of major importance.¹ Many studies have been made using various techniques to investigate the propagation and stability of waves in beam-plasma systems, however, the picture is still not complete. The weaknesses of the existing linear theory can be brought out by a qualitative discussion of some of the work that has been done to date.

First a review of the various equilibrium models that have been used will be presented and then a review of the beam-plasma interactions that have been studied will be presented. The emphasis here will be on waves with frequencies between the ion cyclotron frequency and the electron cyclotron frequency and, in general the electron beam and plasma will be assumed to be cold, although some investigators have treated warm beams.^{2,11} Furthermore, the beam will be assumed to propagate along a uniform magnetic guide field, B_0 , and to be much less dense than the plasma.

Equilibrium Models

Obviously the simplest model for a beam-plasma system consists of an infinite homogeneous cold beam passing through an infinite homogeneous cold plasma. Under these conditions it has been shown that for a beam much less dense than the plasma the dispersion equation can be separated into three sets of waves when $k \parallel B_0$: longitudinal waves, and transverse waves with left and right circular polarization.^{3,4} The purely longitudinal modes

will not be discussed at length here since a summary of these results can be found elsewhere.^{5,6} This model has been used by many investigators in studying the beam-plasma interaction problem for transverse waves, however it is not a very realistic model.^{3,4,6-11,15} Physically the electron beam and plasma must be bounded both radially and axially, although the bound on the axial extent is probably not as crucial. Once finite radial dimensions have been introduced in the problem, it has been shown that the longitudinal and transverse waves are no longer decoupled.^{3,4} For this reason the results of this model could be misleading.

Another weakness of this simple model as applied to very intense electron beams is that it ignores the transverse energy of the beam. In order to improve upon this some authors have used a helical beam or streaming oscillator model, where all the beam electrons gyrate about the magnetic field lines with the same transverse velocity.^{11,17-23,39} Although this is an improvement on the simple model it still suffers from the problem of infinite radial extent.

Treating the case of finite radial size, of course, increases the difficulty of the problem immensely. To simplify matters somewhat, the quasistatic approximation was used by investigators who first attempted to solve the radial problem.^{7,24-28} In this approximation the curl of the electric field is set equal to zero, in which case the theory is only valid for waves with a phase velocity much less than the speed of light. To treat the case of waves with phase velocities near the velocity of light some investigators have now tackled the full electromagnetic problem with radial boundaries.^{4,27,30,31} Comparisons of the full treatment and the quasistatic treatment have revealed that

the dispersion curves calculated from the quasistatic treatment are in error for beam energies in excess of 80 keV.³⁰ Furthermore, the quasistatic treatment incorrectly predicts that the upper pass band of a cold plasma waveguide is of a backward wave nature, whereas the full treatment predicts it may be of a forward nature when $\omega_{ce} > \omega_{pe}$.⁴

In most of the work that has been done to date on the bounded beam-plasma problem the electron beam has been assumed to either fill a waveguide or have a square radial profile. This type of model is simple to work with but is lacking somewhat in physical content, particularly if $k_1 R \lesssim 1$, where k_1 is the radial wavenumber and R is the radius of the beam. Some results have been obtained by averaging over a sharp boundary at the edge of the beam-plasma interface.²⁷ Also some work has been performed on modeling a beam profile by a series of cylindrical electron beam shells.³⁰ In this case there was some indication that the faster growing modes may be suppressed. However this is still an area where there is much to be learned.

One final approximation that has been used on occasion consists of assuming that the applied magnetic field is very large.^{7,32} This approximation obviously ignores all cyclotron effects that would occur naturally.

Of course there are situations that arise where any one of these models will be appropriate but it seems that to thoroughly explore the problem of heating a plasma by an intense relativistic electron beam, the full electromagnetic treatment including radial dependence should be used. This is particularly true for relativistic beams since a large γ enhances the coupling between the transverse and longitudinal modes. This coupling is even further strengthened by the presence of

self magnetic fields and beam rotation which have received little attention in the literature at this point.

Beam-Plasma Interactions

A beam-plasma interaction may occur whenever waves which propagate on the electron beam occur in the same region of ω - k space as waves that propagate in the plasma. Since it was assumed that the beam density is much less than the plasma density, the plasma waves are essentially unaffected by the presence of the beam except at these isolated points of interaction. At these points the plasma waves are said to be in synchronism with the beam. Essentially these interactions correspond to resonances between plasma waves and the longitudinal or transverse motion of the beam electrons. The beam waves involved in the resonances with the longitudinal motion of the beam electrons are the fast and slow beam space charge waves, and the beam waves involved in the resonances with the transverse motion of the beam electrons are the fast and slow beam cyclotron waves. Figure 1 gives a ω - k plot of these beam waves for the case of an infinite homogeneous cold beam. Since electrons rotate in a right hand sense about the magnetic guide field, strong beam cyclotron interactions should occur with right hand polarized plasma waves at points marked by a + on Figure 1 and weak interactions should occur at points marked by a -.

When an unstable interaction occurs such that the phase velocity of the wave is equal to the velocity of the beam, $V_\phi = V_b$, it is considered to be due to the Vavilov-Cherenkov effect. When $V_\phi < V_b$, then the instability is due to the anomalous Doppler effect and when $V_\phi > V_b$ the instability is due to the normal Doppler effect.⁶

Instabilities due to the normal Doppler effect are only possible if the beam electrons possess transverse energy.^{33,21} However, in general an instability is simply identified by naming the beam wave and plasma wave involved in the interaction.

Another classification for the different beam-plasma instabilities involves classifying the type of medium in which the wave is propagating. A reactive medium instability occurs when the dielectric constant becomes negative and a resistive medium instability occurs when a negative energy wave grows as it loses energy due to Landau damping.⁷

Once an interaction is identified it must be determined whether or not the wave is evanescent, convectively unstable or absolutely unstable. Methods have been worked out for making this determination by investigating the poles of the dispersion equation.^{7,34,35} Obviously a convective instability will have little importance if it convects out of the system before it grows to an appreciable level. Similarly, an absolute instability is ignorable if it grows on a time scale much slower than the time scale of the experiment. The significance of the time and length scales of the beam-plasma system are discussed in greater detail by a few authors.^{7,34,35}

Consider now the unstable interactions that have been studied using the infinite homogeneous cold beam plasma model. For propagation exactly along the magnetic guide field, the longitudinal waves, the right circularly polarized waves and the left circularly polarized waves are all uncoupled. Under these conditions there are few beam-plasma interactions. The interaction with the largest growth occurs when a plasma Langmuir wave is in synchronism with a beam space charge wave.^{7,9,11,12,14} This is merely the well known two-stream

instability.* There is also an interaction between the slow beam cyclotron wave and the plasma ion cyclotron wave as well as other lower frequency instabilities but these are all below the frequency range of interest.^{7,8,9}

If the beam possesses transverse energy as well as the streaming energy along the applied field, then even for the case of $k \parallel B_0$ the beam is capable of driving additional plasma waves unstable when the cyclotron resonance condition is satisfied, $\omega - k_z V_b + \omega_{ce} = 0$.^{11,21,33} In fact, when k is not parallel to the magnetic field, the growth rates of the cyclotron instabilities discussed below should also be modified when the beam possesses transverse energy.¹¹

If the perpendicular wavenumber is allowed to be nonzero, then some of the effects of finite radial size can be simulated. This procedure has been carried out using both the quasistatic approximation^{8,16,27} and the full electromagnetic treatment,^{9,16} however analytic expressions for the growth rates are only available from the electrostatic analysis. Here the longitudinal and transverse waves become coupled leading to two new interactions. For $\omega_{pe} > \omega_{ce}$ the first appears when a wave associated with the plasma cyclotron wave branch becomes synchronous with a beam space charge wave. The other interaction occurs between a plasma Langmuir wave and a slow beam cyclotron wave. Both these interactions are unstable and expressions for their quasistatic growth rates can be found in Table 1. In addition to these a third new instability arises since the transverse waves no longer split nicely into right and left circularly polarized waves. This third new instability involves

* Many references for this instability may be found in Refs. 5 and 6.

a wave associated with the plasma cyclotron wave branch, and a slow beam cyclotron wave. Its growth rate can also be found in Table I. Finally, the two-stream instability becomes dependent on the angle of propagation as indicated in Table I since the wave is no longer traveling exactly along the magnetic guide field. Figure 2 shows the position of these interactions on a ω - k diagram, and the labels referring to each instability are listed in Table I for comparison.

In order to facilitate comparisons, Table I lists expressions for the growth rates of these four instabilities in three different parameter regimes. The quasistatic approximation ($\omega/k \ll c$) was used in deriving all of these expressions so that the results for the Cherenkov instabilities ($\omega = k_z v_b$) should be used cautiously when the beam is relativistic. For a nonrelativistic beam the expressions are correct with $\gamma_b = 1$. The numerical factors η_{cher} and η_c are both equal to unity for the infinite beam plasma problem. If the plasma fills a cylindrical waveguide of radius R and the beam propagates along the axis with a radius r ($r \leq R$), then²⁷

$$\eta_{\text{cher}} = \frac{r^2}{R^2} \frac{J_n^2(x_{nv} \frac{r}{R}) - J_{n-1}(x_{nv} \frac{r}{R}) J_{n+1}(x_{nv} \frac{r}{R})}{J_n'^2(x_{nv})}$$

and

$$\eta_c = \frac{r^2}{R^2} \frac{J_{n+1}^2(x_{nv} \frac{r}{R}) - J_n(x_{nv} \frac{r}{R}) J_{n+2}(x_{nv} \frac{r}{R})}{J_n'^2(x_{nv})}$$

where x_{nv} is the v th root of the n th order Bessel function. Note that if the beam also fills the waveguide ($r=R$) then again $\eta_{\text{cher}} = \eta_c = 1$. For the infinite beam plasma case θ is the angle between the wave vector, k , and the axial magnetic field, however for the bounded beam plasma

case θ is defined by²⁷

$$\cos \theta = \frac{k_z R}{\sqrt{k_z^2 R^2 + x_{nv}^2}}$$

It should be pointed out that aside from the definition of the angle θ , the dispersion relations in the quasistatic limit for an infinite or bounded magnetoactive plasma in the absence of the beam are identical, with the two roots given by²⁷

$$\omega_{\pm}^2 = \frac{1}{2} (\omega_p^2 + \omega_c^2) \left(1 \pm \sqrt{1 - \frac{4\omega_p^2 \omega_c^2 \cos^2 \theta}{(\omega_p^2 + \omega_c^2)^2}} \right).$$

Here ω_p is the electron plasma frequency and ω_c is the electron cyclotron frequency.

In deriving the results of Table I^{8,16,27} it was assumed that $\delta/\omega \ll 1$ and $\delta/\omega_R \ll 1$ where δ is the growth rate of any instability with frequency ω and where $\omega_R = \omega_c/\gamma_b$. These conditions are easily satisfied if $\gamma_b \left(\frac{n_b}{n_p}\right)^{1/3} \ll 1$ when $\omega_c \gtrsim \omega_p$, and for instabilities a and d when $\omega_c \ll \omega_p$. For instabilities b and c with $\omega_c \ll \omega_p$ the above conditions are only satisfied if $\frac{n_b}{n_p} \ll \frac{\omega_c^3}{2\gamma_b \omega_p^3}$ for instability b and $\frac{n_b}{n_p} \ll \frac{\omega_c^3}{3\omega_p^3}$ for instability c. If, however, we replace the above conditions with the condition that

$$\omega \sim \omega_p \gg \delta \gg \omega_R$$

then the growth rates of instabilities b and c are shown in Table II and are valid for $\left(\frac{n_b}{n_p}\right)^{1/3} \ll 1$. Note that in this case these two instabilities have essentially coalesced since the magnetic field was assumed very weak, implying that

$$\omega - k_z v_b + \omega_R = \omega - k_z v_b.$$

For this reason the growth rates are identical.

The most important result that is obtained from the information in these tables is that in certain parameter ranges the growth rate of the two-stream instability is not the largest growth rate. When $\omega_c \gg \omega_p$ the growth rates in Table I are valid if $\left(\frac{n_b}{n_p}\right)^{1/3} \ll 1$. In this case if $1 \gg \left(\frac{n_b}{n_p}\right)^{1/3} > \frac{9\sqrt{3}}{4\gamma_b^2} \frac{\eta_{\text{cher}}^{2/3}}{\eta_c}$ then the growth rate of cyclotron interaction A is largest. Although it has not been pointed out in the literature, in the very weak relativistic beam case $\left(\frac{n_b}{n_p} \ll \frac{\omega_c^3}{\gamma_b^2 \omega_p^3}\right)$, $\gamma_b^2 \gg 1$ when $\omega_p \gg \omega_c$, the cyclotron interaction b also has the largest growth rate if $1 \gg \left(\frac{2}{\gamma_b} \frac{n_b}{n_p}\right)^{1/3} \frac{\omega_p}{\omega_c} > \left(\frac{27}{4\gamma_b^4}\right)^{1/3} \frac{\eta_{\text{cher}}^{2/3}}{\eta_c}$. Note that this instability has its maximum growth rate for propagation exactly perpendicular to the magnetic guide field as does the Weibel instability. However the growth rate of the Weibel instability¹⁶ is only

$$\delta^w = \left[\left(\frac{v_b}{c}\right)^2 \left(\frac{n_b}{\gamma_b n_p}\right) \left(\frac{k_c^2}{k_c^2 + \omega_p^2}\right) \omega_p^2 - \frac{\omega_c^2}{\gamma_b^2} \right]^{1/2},$$

which is less than the growth rate of interaction b by a factor of $\left(\frac{\omega_p}{\omega_c}\right)^{1/2}$. The Weibel instability also stabilizes for $\frac{\omega_c}{\gamma_b} > \frac{v_b}{c} \omega_p \left(\frac{n_b}{\gamma_b n_p}\right)^{1/2}$, so that for very weak relativistic beams interaction b will be the most important instability when the magnetic field is not too small ($\delta/\omega_R \ll 1$).

Numerical solutions of the full electromagnetic dispersion equations have been computed for the case of an infinite homogeneous beam plasma system.^{9,16} The spectrum in k_z space tended to be wider for the Cherenkov instabilities than for the cyclotron instabilities. In a

strong magnetic field the growth rate as a function of angle of the linear electron two-stream instability was shown to be narrower in angle than the results found using an electrostatic treatment.

Essentially the two-stream instability becomes one dimensional in the direction along the applied field in the strong field case. This is in sharp contrast to the weak magnetic field case, where the two-stream instability has a maximum growth rate for waves propagating perpendicular to the magnetic field. When $\omega_{ce} > \left(\frac{\gamma n_b}{2n_p}\right)^{1/2} \omega_{pe}$, it was also found that the growth rates of plasma waves driven by an interaction with the slow beam cyclotron wave become independent of the beam energies.

Since all the longitudinal and transverse waves are coupled in the radially bounded problem, it would be reasonable to assume that the four instabilities discussed above will carry over to the bounded problem, and, in fact, they do. The most interesting consequence of these findings is that the analog of the two-stream instability is not necessarily the only important instability. It is now necessary to investigate all four of these instabilities, and possibly more, in order to determine the mechanisms involved in heating the plasma.

The quasistatic approximation was employed in the first attempts to solve the radially bounded beam plasma problem.^{7,24-28} Using this approximation the waves that propagate in a plasma filled waveguide differ somewhat from the plasma waves found in the unbounded case. Note from Figure 3 that the character of the wave changes depending on whether ω_{ce} or ω_{pe} is larger, and that the upper pass band is a backward wave in this electrostatic model. Figure 4 indicates the dependence of the waves on the radial wavenumber in the quasistatic approximation.³⁷

Figure 5 shows the beam modes superimposed over the plasma filled waveguide modes and indicates the position of the four interactions. Note that the interaction between the slow beam cyclotron wave and the backward wave is an absolute instability while the other three interactions are all convective instabilities. It has been found that the interactions extend into regions off resonance but that the largest growth rates occur when the plasma waves are in synchronism with the beam. Furthermore, the high frequency instabilities tend to be dominant and high electron thermal energies are necessary to obtain a strong interaction with the ions.⁷ This is true since the electrons tend to short out any low frequency electric fields.

The quasistatic approximation is only valid for waves with phase velocities much less than the speed of light, hence it was found that the approximation is not appropriate when treating relativistic electron beams, particularly when treating Cherenkov interactions.³⁰ It has also been shown that the backward wave in Figure 3 may be a forward wave for $\omega_{ce} > \omega_{pe}$ when the quasistatic approximation is not used.⁴ When the full electromagnetic treatment is employed the same four interactions are present, however many new results are found.^{4,26,30,31} In Figure 5c the lines marked B correspond to gyrating beam electrons that are nearly completely decoupled from the plasma, and the curves P and Q correspond to plasma-filled waveguide modes. Neither of these features of the dispersion curves are present in the quasistatic calculations in Figure 5b.³⁰

If the ion dynamics are ignored and the beam is treated non-relativistically, the full electromagnetic theory for a radially bounded system has shown that the quasistatic treatment is in

fair agreement if $\omega_{pe} > \omega_{ce}$.³¹ However when $\omega_{ce} > \omega_{pe}$ and $\frac{c}{R\omega_{ce}} < 1$, the two theories differ considerably in the upper plasma modes. Here R is the radius of the cylindrical waveguide. The backward wave predicted by quasistatic theory transforms into a forward wave under these conditions, and thus the nature of the interactions between the wave and the beam changes considerably. The interaction of the plasma lower pass band wave with the fast beam cyclotron wave which previously resulted in an evanescent wave now yields two normal propagating waves. The three-wave interaction of the plasma lower pass band wave with the fast and slow beam space charge waves now splits into two two-wave interactions, and finally coupling with empty waveguide modes becomes possible, in contrast to the quasistatic treatment.

When ion dynamics are included and the beam is treated relativistically it was found from numerical calculation of the dispersion curves that the quasistatic approximation was not valid if the beam energy exceeded 80 keV and the radial wave number was not too small.³⁰ Figure 6 clearly shows the discrepancies between the dispersion curves predicted by the two theories. Since it was shown that high beam energies are necessary to deposit a large amount of energy in the ions, the full electromagnetic treatment must be used to explore ion heating.³⁰ In fact, for large values of the ratio $\frac{k_{\perp}}{k_{\parallel}}$, the interaction between the lower plasma pass band and the fast beam space charge wave theoretically produces ion energies comparable with the electron energies involved when $\frac{\omega_{ce}}{\omega_{pe}} \sim 1$. Since the instabilities associated with the upper plasma pass band (see Figure 3) were shown to have higher growth rates, modulation of the beam was suggested to enhance the growth of the desired mode. In one example it was calculated that the

ions attained energies on the order of 400 eV before nonlinear effects limited the growth of the instability when an electron beam of 13.6 A and 80 keV was used.³⁰

Summary

In summary there are at least four instabilities that are important in the radially bounded beam-plasma problem. It seems that a full electromagnetic treatment including relativistic effects is required in order to properly describe the mechanisms involved in heating the plasma ions. However very little work has been done on considering the effects of self magnetic fields or beam rotation although there is experimental evidence suggesting that it may be important.^{38,39} Another area which also warrants investigation is the effects of the radial beam profile. In general, the beam is assumed to fill the entire waveguide or to have a square profile which does not give realistic results when the radial wavelength is of the same order as the radius of beam or larger ($k_{\perp} R \lesssim 1$).

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PARAMETER RANGE	EFFICIENCY	WAVELENGTH	GROWTH RATE (6)	LABEL (FIG. 2 & 5b)
$\omega_c \gg \omega_p$	$\omega^2 = \omega_c^2 + \omega_p^2 \sin^2 \theta$	$\omega = k_z v_b$	$\frac{\sqrt{3}}{2\gamma_b} \omega_p \left[\eta_{\text{cher}} \frac{n_b}{2n_p} \frac{\omega_p}{\omega_c} \sin^2 \theta \cos^2 \theta \right]^{1/3}$	C
	$\omega^2 = \omega_p^2 \cos^2 \theta$	$\omega = k_z v_b$	$\frac{\sqrt{3}}{2\gamma_b} \omega_p \left[\eta_{\text{cher}} \frac{n_b}{2n_p} \cos^2 \theta \right]^{1/3}$	D
	$\omega^2 = \omega_c^2 + \omega_p^2 \sin^2 \theta$	$\omega = k_z v_b - \frac{\omega_c}{\gamma_b}$	$\frac{\omega_p}{2\omega_c} \sin^2 \theta \left(\eta_c \frac{n_b}{n_p} \right)^{1/2}$	B
	$\omega^2 = \omega_p^2 \cos^2 \theta$	$\omega = k_z v_b - \frac{\omega_c}{\gamma_b}$	$\frac{\omega_p \sin^2 \theta}{2} \left(\eta_c \frac{n_b}{n_p} \frac{\omega_p}{\omega_c} \cos^2 \theta \right)^{1/2}$	A
$\omega_c = \omega_p$	$\omega^2 = \omega_c^2 (1 + \sin^2 \theta)$	$\omega = k_z v_b$	$\frac{\sqrt{3}}{2\gamma_b} \omega_c (1 + \sin^2 \theta)^{1/6} \left[\eta_{\text{cher}} \frac{n_b}{4n_p} \cos^2 \theta \right]^{1/3}$	
	$\omega^2 = \omega_c^2 (1 - \sin^2 \theta)$	$\omega = k_z v_b$	$\frac{\sqrt{3}}{2\gamma_b} \omega_c (1 - \sin^2 \theta)^{1/6} \left[\eta_{\text{cher}} \frac{n_b}{4n_p} \cos^2 \theta \right]^{1/3}$	

TABLE I (CONTINUED ON NEXT PAGE)

PARAMETER RANGE	FREQUENCY	WAVENUMBER	GROWTH RATE (δ)	LABEL (FIG. 2 & 5b)
$\omega_c = \omega_p$	$\omega^2 = \omega_c^2 (1 + \sin^2 \theta)$	$\omega = k_z v_b - \frac{\omega_c}{\gamma_b}$	$\frac{\omega_c}{\sqrt{2}} (1 + \sin^2 \theta)^{1/4} \sin \theta \left(\frac{n_b}{n_c n_p} \right)^{1/2}$	
	$\omega^2 = \omega_c^2 (1 - \sin^2 \theta)$	$\omega = k_z v_b - \frac{\omega_c}{\gamma_b}$	$\frac{\omega_c}{\sqrt{2}} (1 - \sin^2 \theta)^{1/4} \sin \theta \left(\frac{n_b}{n_c n_p} \right)^{1/2}$	
$\omega_c \ll \omega_p$	$\omega^2 = \omega_p^2 + \omega_c^2 \sin^2 \theta$	$\omega = k_z v_b$	$\frac{\sqrt{3}}{2 \gamma_b} \omega_p \left[\eta_{\text{cher}} \frac{n_b}{2 n_p} \cos^2 \theta \right]^{1/3}$	c
	$\omega^2 = \omega_c^2 \cos^2 \theta$	$\omega = k_z v_b$	$\frac{\sqrt{3}}{2 \gamma_b} \omega_c \cos \theta \left(\eta_{\text{cher}} \frac{n_b}{2 n_p} \sin^2 \theta \right)^{1/3}$	d
	$\omega^2 = \omega_p^2 + \omega_c^2 \sin^2 \theta$	$\omega = k_z v_b - \frac{\omega_c}{\gamma_b}$	$\frac{\omega_p}{2} \sin \theta \left(\frac{n_b}{n_c n_p} \frac{p}{\omega_c} \right)^{1/2}$	b
	$\omega^2 = \omega_c^2 \cos^2 \theta$	$\omega = k_z v_b - \frac{\omega_c}{\gamma_b}$	$\frac{\omega_c}{2} \sin^2 \theta \left(\frac{n_b}{n_c n_p} \cos \theta \right)^{1/2}$	a

TABLE I - GROWTH RATES (QUASISTATIC APPROX.)

FOR $\delta \ll \omega$ AND $\delta \ll \omega_c / \gamma_b$

PARAMETER RANGE	FREQUENCY	WAVENUMBER	GROWTH RATE (δ)	LABEL (FIG. 2)
$\omega_c \ll \omega_p$	$\omega^2 = \omega_p^2 + \omega_c^2 \sin^2 \theta$	$\omega = k_z v_b$	$\frac{\sqrt{3}}{2} \omega_p \left(\frac{n_b}{2\gamma_b n_p} \right)^{1/3} \left(\sin^2 \theta + \frac{\cos^2 \theta}{\gamma_b} \right)^{1/3} \tau_{\text{cher}}^{1/3}$	c
	$\omega^2 = \omega_c^2 \cos^2 \theta$	$\omega = k_z v_b$	$\frac{\sqrt{3}}{2\gamma_b} \omega_c \cos \theta \left\{ \eta_{\text{cher}} \frac{n_b}{2n_p} \sin^2 \theta \right\}^{1/3}$	d
	$\omega^2 = \omega_p^2 + \omega_c^2 \sin^2 \theta$	$\omega = k_z v_b - \frac{\omega_c}{\gamma_b}$	$\frac{\sqrt{3}}{2} \omega_p \left(\frac{n_b}{2\gamma_b n_p} \right)^{1/3} \left(\sin^2 \theta + \frac{\cos^2 \theta}{\gamma_b} \right)^{1/3} \tau_c^{1/3}$	b
	$\omega^2 = \omega_c^2 \cos^2 \theta$	$\omega = k_z v_b - \frac{\omega_c}{\gamma_b}$	$\frac{\omega_c}{2} \sin^2 \theta \left\{ \eta_c \frac{n_b}{n_p} \cos \theta \right\}^{1/2}$	a

TABLE II - GROWTH RATES (QUASISTATIC APPROX.)

$\delta/\omega \ll 1$ and $\delta \gg \frac{\omega_c}{\gamma_b}$ (WEAK MAGNETIC FIELD CASE)

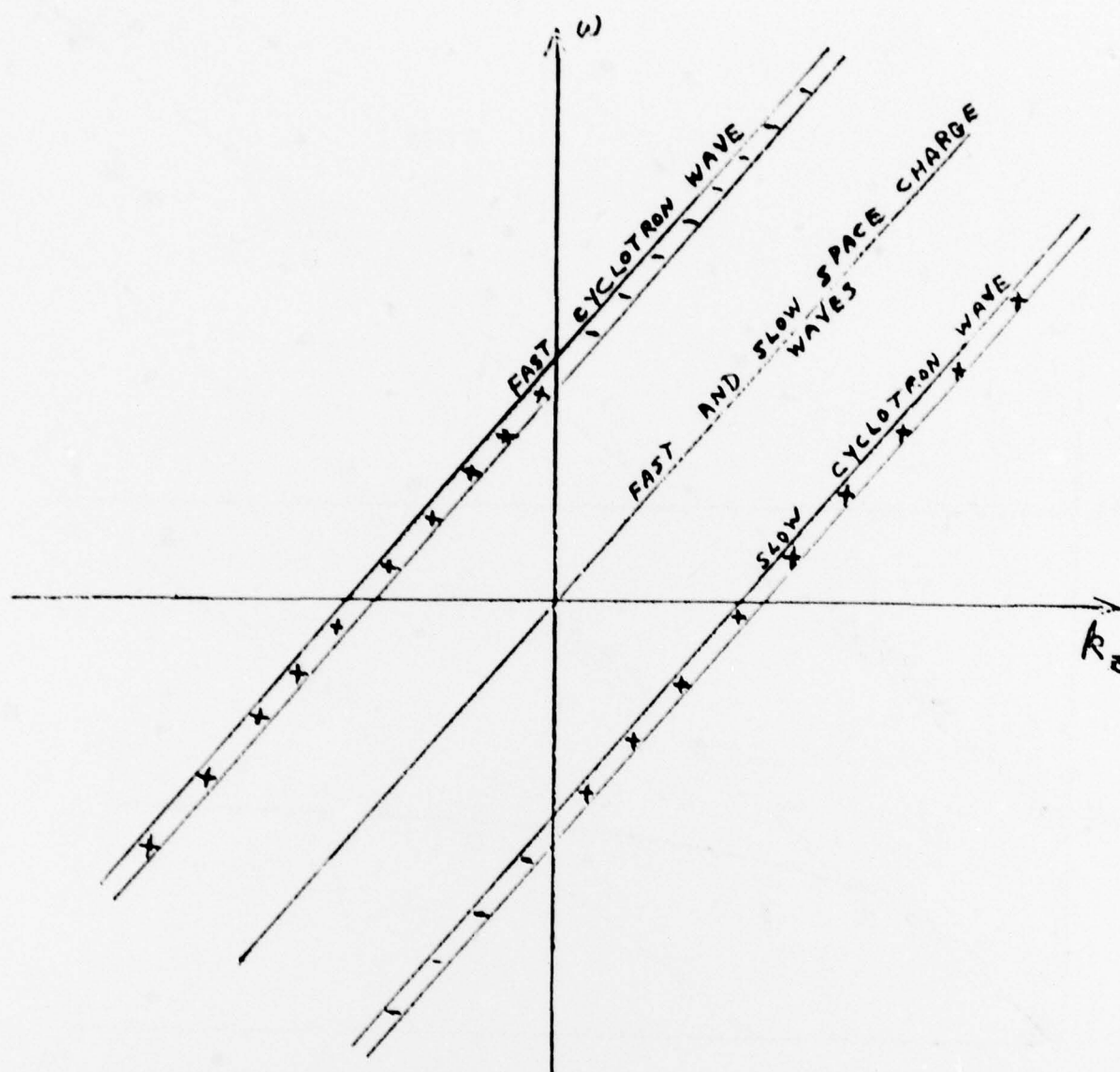


FIGURE 1 - ω - k_z PLOT OF BEAM WAVES ON AN INFINITE
HOMOGENEOUS ELECTRON BEAM [AFTER KNOX, C.F.,
PLASMA PHYS. 1, 1 (1967).]

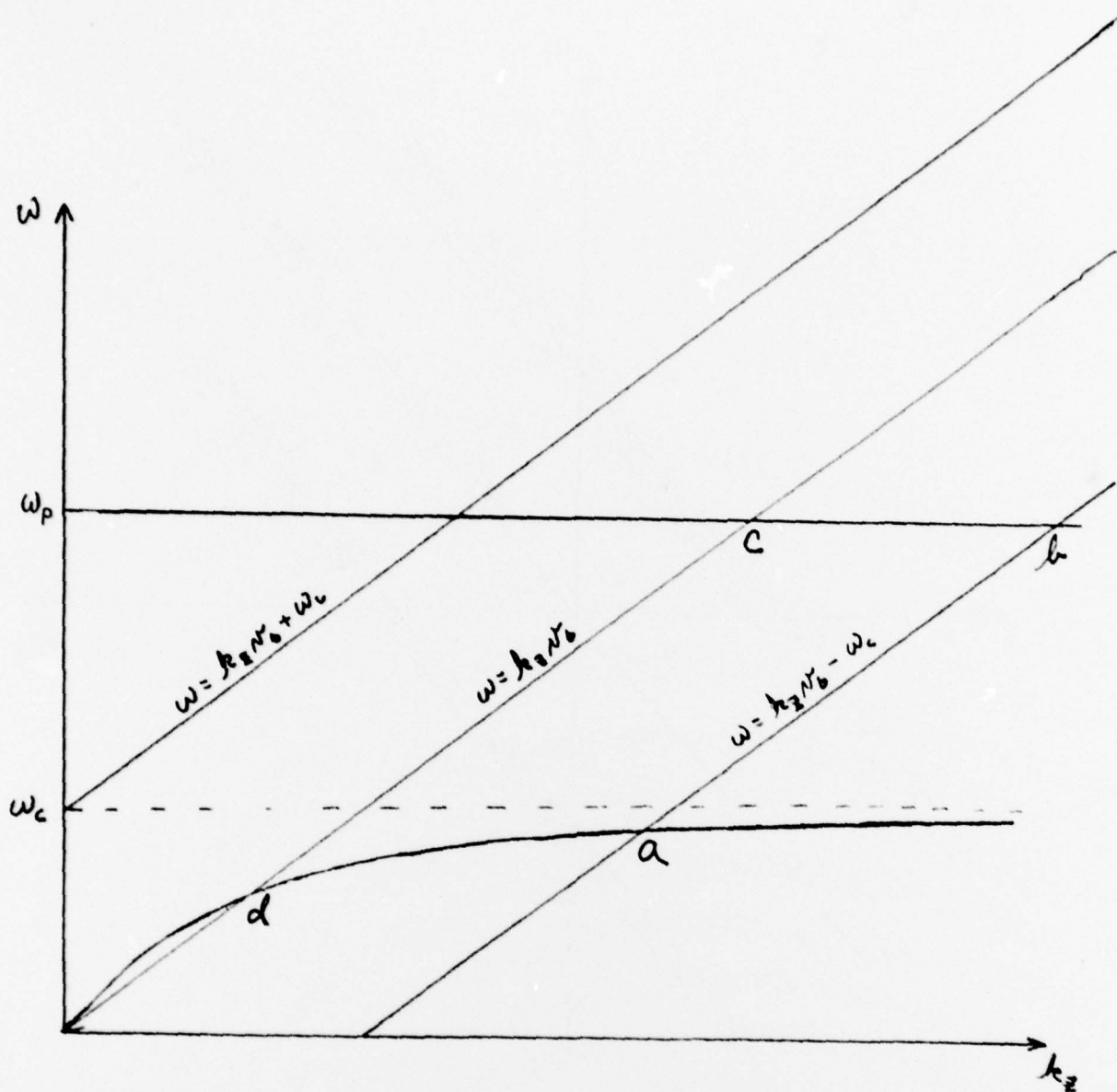


FIGURE 2 - POSITION OF THE FOUR BEAM
PLASMA INTERACTIONS FOR $\left| \frac{k_{\perp}}{k_z} \right| \gtrsim 0$

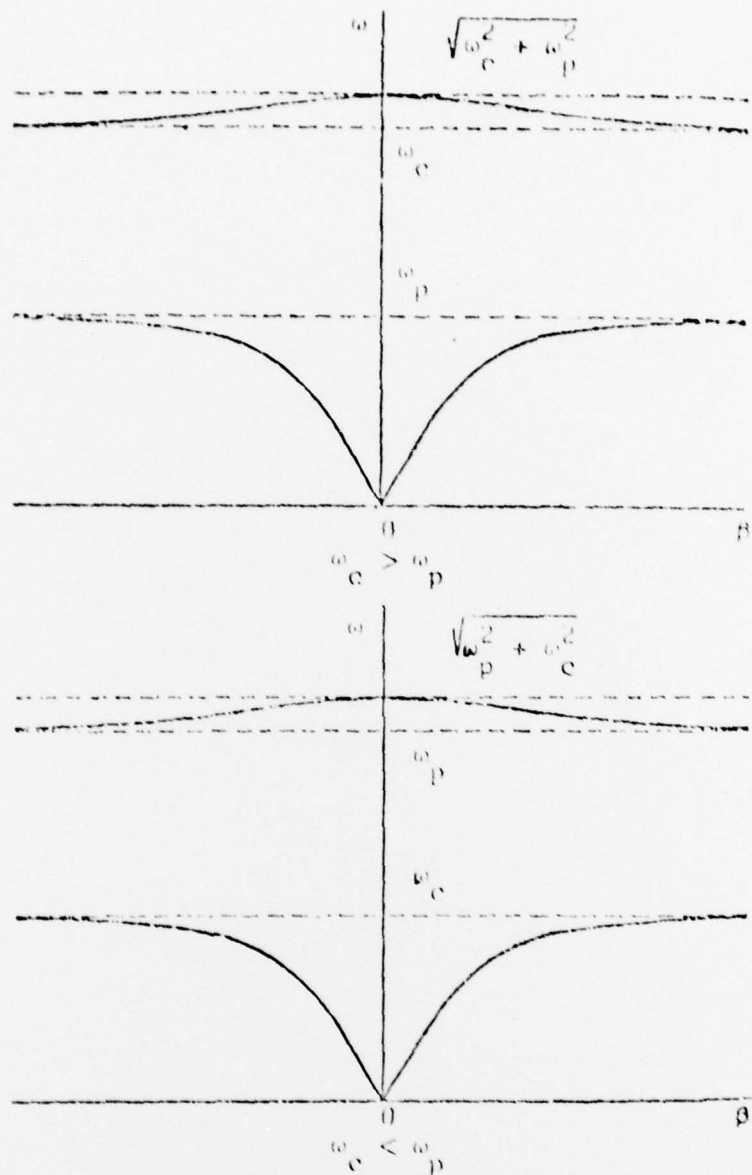


FIGURE 3 - PHASE CHARACTERISTIC OF ELECTROSTATIC WAVES IN A PLASMA-FILLED WAVEGUIDE. β IS THE AXIAL WAVENUMBER. [FROM TRIVELPIECE, A. W., SLOW WAVE PROPAGATION IN PLASMA WAVEGUIDES, SAN FRANCISCO PRESS INC., SAN FRANCISCO (1967).]

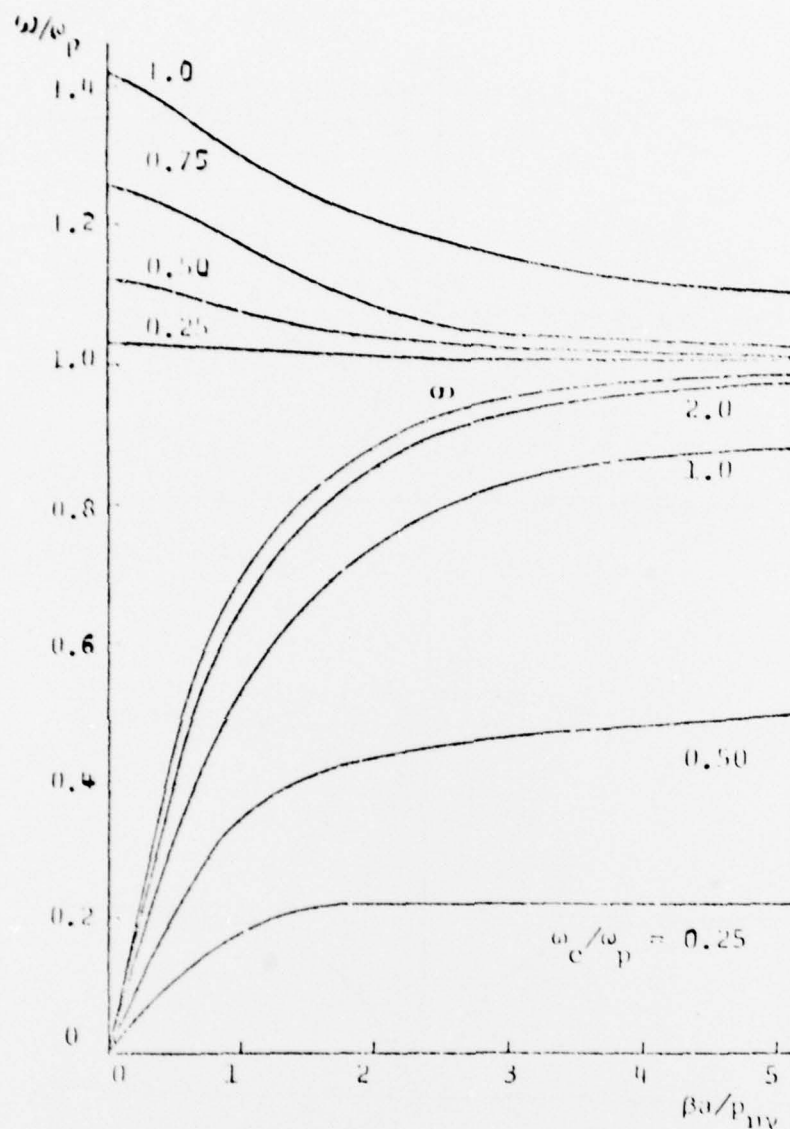


FIGURE 4 - PHASE CHARACTERISTIC OF A PLASMA-FILLED WAVEGUIDE OF RADIUS a . p_{nv} IS THE v th ZERO OF THE n th ORDER BESSEL FUNCTION OF THE FIRST KIND, AND β IS THE AXIAL WAVENUMBER [FROM TRIVELPIECE, A. W., SLOW WAVE PROPAGATION IN PLASMA WAVEGUIDES, SAN FRANCISCO PRESS INC., SAN FRANCISCO (1967).]

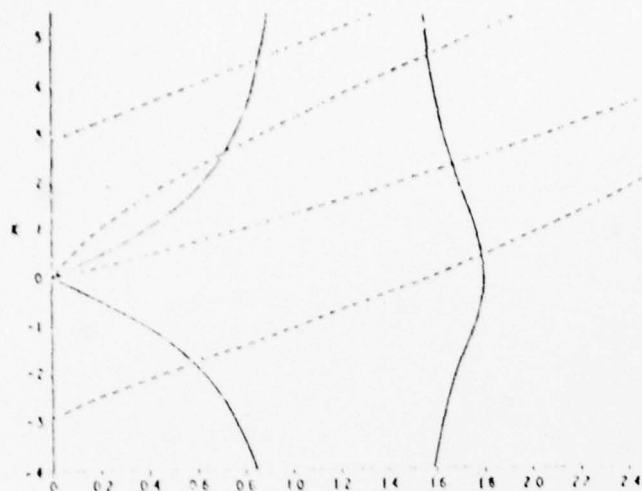


FIG. 5a.

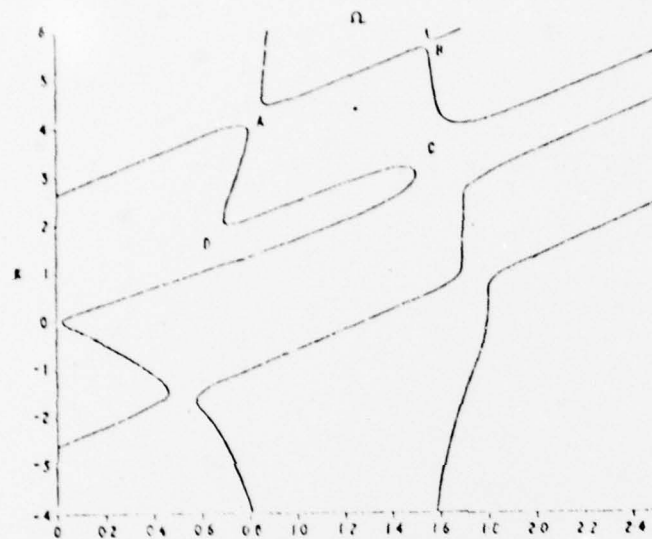


FIG. 5b.

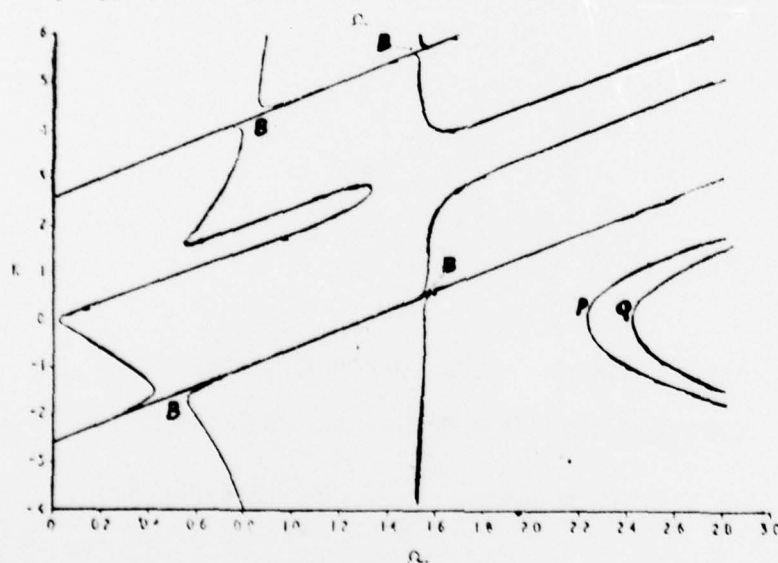


FIG. 5c.

FIGURE 5 - a) SEPARATE DISPERSION CURVES FOR PLASMA ELECTRONS (SOLID LINES) AND BEAM ELECTRONS (DASHED LINES). DISPERSION CURVES CALCULATED USING b) THE QUASISTATIC APPROXIMATION AND c) THE GENERAL SOLUTION. ALL CASES FOR

$$\frac{n_b}{n_p} = .1, \frac{\omega_c}{\omega_p} = 1.5, \beta = 0.5, \frac{k_c}{\omega_p} = 2.0 \text{ [FROM ADLAM, J.H., PLASMA PHYS. 13,}$$

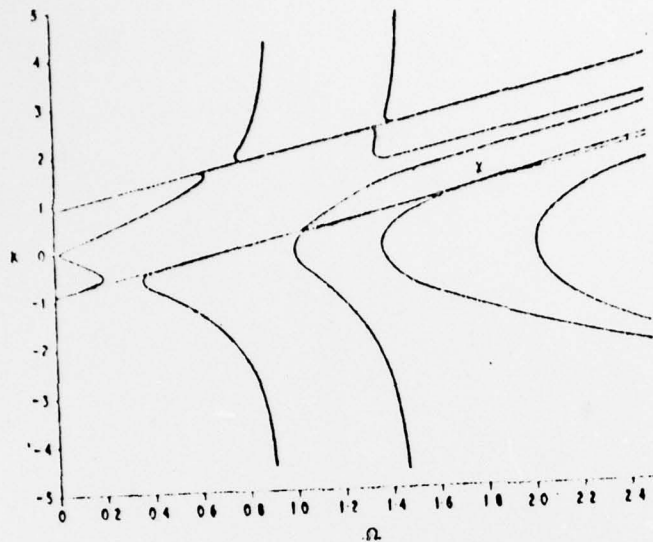


FIG. 6a.

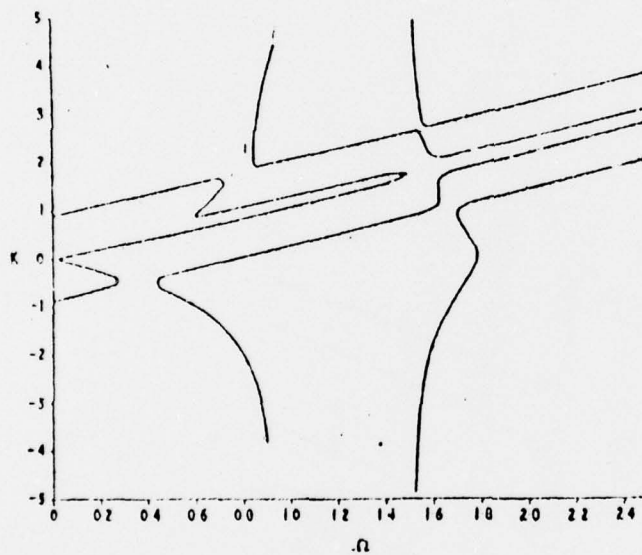


FIG. 6b.

FIGURE 6 - DISPERSION CURVES CALCULATED USING a) THE GENERAL SOLUTION b) THE QUASISTATIC APPROXIMATION. BOTH CASES FOR $\frac{n_b}{n_p} = 0.1$, $\frac{\omega_c}{\omega_p} = 1.5$, $\beta = 0.86324$, $\frac{k_1 C}{\omega_p} = 1.0$, BEAM ENERGY = 500 keV. [FROM ADLAM, J. H., PLASMA PHYS. 13, 329 (1971).].